Self-organised systems: fundamental properties

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Abstract

A set of fundamental properties of self-organised systems is identified. Asynchronism is here proposed as one of these properties. It is shown that, by overlooking it, the concept of self-organisation is not fulfilled. Implications of this property to the study of self-organisation are discussed. Further, two other salient aspects are identified: minimisation of local conflicts produces optimal evolutionarily stable self-organisation; and the hypothesis that complexity variations may distinguish living from non-living self-organised systems. Conclusions and further research bring the document to an end.

Keywords: Self-organisation, asynchronism, complexity, Cellular Automata, evolutionarily stable strategies.

Resumo

Neste artigo identifica-se um conjunto de propriedades fundamentais que caracterizam os sistemas auto-organizados. Propõe-se o assincronismo como uma dessas propriedades. Mostra-se que sistemas sem assincronismo não se podem considerar auto-organizados. Discutem-se as implicações desta propriedade para o estudo da auto-organização. Para além disso, identificam-se dois outros aspectos notáveis: a minimização de conflitos locais produz auto-organização óptima e evolucionariamente estável; e a hipótese de que as variações de complexidade permitem distinguir sistemas auto-organizados vivos dos não vivos. O documento termina com as principais conclusões e perspectivas de investigação futura.

Palavras-chave: Auto-organisação, assincronismo, complexidade, Autómatos Celulares, estratégias evolucionariamente estáveis.

1- Introduction

Self-organised systems are ubiquitous whether in physics, chemistry, biology, sociology, economy or engineering [Bettstetter et al., 2005, Camazine et al., 2001, Foster, 2000, Fuchs, 2003, Normand et al., 1977, Parrish and Edelstein-Keshet, 1999, Ryterman and Recanatini, 2001, Serugendo et al., 2006]. As an informal description, a self-organised system is composed of several elements and autonomously modifies its structure to display more coherent behaviours. Kauffman [Kauffman, 2005] has even suggested that self-organisation is a basic property of nature, over which selection operates, that led to the appearance of life on Earth. We can find self-organisation in convection phenomena, in insect colonies, pedestrian traffic, or Ethernet, among others.

In spite of its presence in a vast diversity of areas and the research dedicated to it, self-organisation is still a subject of controversy in terms of a precise definition [Gershenson and Heylighen, 2003]. Attempts at mathematical definitions consider it to show an increase in complexity [Shalizi and Shalizi, 2005], or an increase in order, which are also concepts subject to a diversity of definitions [Feldman and Crutchfield, 1998].

Here we address a discussion on the definition of self-organisation based on its most significant properties. From commonly agreed ideas found in the literature and in previous work [Correia, 2006], we arrive at a set of five fundamental properties. This is done by analysing different types of embodied self-organised systems. One of those properties, asynchronism, is here originally presented. Besides the global defining properties, we analyse a few aspects of self-organised behaviour, taken as consequences of the fundamental properties.

In the next section we present the fundamental properties of self-organisation and add a few comments on their importance. Next, a few consequences of these properties are described. The document concludes with a short recapitulation of the main results and presents ideas for future research.

2- Defining self-organisation by its fundamental properties

A formal definition of self-organisation still seems to be elusive. However, we may approach it by describing the fundamental properties of self-organisation. If any of these properties is absent a system is not self-organised. Before proceeding it is worth noting that self-organisation and emergence are two distinct concepts, but frequently confused. From a table in [Anderson, 2002] quoting ten definitions by different authors, we clearly notice, in several cases, a mix-up between self-organisation and emergence.

A comparative analysis of self-organisation and emergence is found in [De Wolf and Holvoet, 2005] and, with an orientation towards Multi-Agent Systems (MAS), in [Serugendo et al., 2006]. We may find and conceive systems with emergence but no self-organisation and the other way around. There are also systems exhibiting both self-organisation and emergence, which may be the source of confusion in attempts to define each of those terms.

In short (see [De Wolf and Holvoet, 2005] for details) fundamental properties of emergence, distinct from the self-organisation case are: *Novelty of global behaviour* - the behaviour observed at a global scale is new compared to the behaviour observed at the individual component, and to its specification; *Decentralised control and interaction* - this means that there is no entity specifying the global behaviour. Components, however, may be controlled, but the global (emergent) behaviour results from their interaction; and *Fault tolerance* - the failure of a single component will not prevent emergence of global behaviour. Multiple component failures will result in a graceful degradation of this behaviour.

We now detail the fundamental properties distinctive of self-organised systems.

2.1- No external control

Although a system may interact with the environment and therefore receive input from it, this input may not be in the form of a behaviour template or guiding control. It may merely be composed of signals that do not specify behaviour and to which the system reacts.

The system reaction is then autonomous, resulting from internal component specification. Therefore, the organisation of the system may not be externally specified. This does not exclude all interactions with the environment and so the system will possibly modify its organisation in reaction to one or more environmental parameters, or *external cues*, as coined in [Parrish and Edelstein-Keshet, 1999]. Nevertheless, these parameters must not contain or express any specific semantics about the system's organisation.

2.2- Increase in order

The system will increase its order as a result of self-organisation. However, order may not increase indefinitely nor does the system have to stay in that higher order configuration thereafter. Order may sometimes decrease as a result of intrinsic or extrinsic factors. For instance, a self-organised fish school will eventually break apart.

Bennett [Bennett, 1988], proposed that self-organisation is the same as spontaneous increase in organisation. We consider this to be a necessary condition, but not sufficient. It overgeneralises and does not clarify the meaning of "spontaneous". An increase in complexity has also been proposed as an alternative, especially because a direct relation between complexity and organisation may be established [Shalizi and Shalizi, 2005].

Definition of complexity is in itself a source of debate. "The more complex the organism, the more difficult it is to predict its behaviour" [Pfeifer and Scheier, 1999], is a good non-formal description of what complexity means. For operational purposes we may consider as a complexity measure the *excess entropy* [Shalizi et al., 2004]:

$$E=\sum_{L=1}^{\infty}[h_\mu(S^L)-h_\mu], \qquad (1)$$

where $h_\mu(S^L)$ is the difference between entropies of two sequences of variables describing the process, such as system states along time, $h_\mu(S^L)=H(S_1,S_2,\dots,S_L)-H(S_1,S_2,\dots,S_{L-1})$, and h_μ is the entropy rate, $h_\mu=\lim_{L\to\infty}h_\mu(S^L)$. Excess entropy measures memory needed about the past sequence to predict future values, which is reasonable as a measure of complexity.

According to property in 2.1 any increase in order must be accomplished autonomously by the system. The system may do it as reaction to an external signal though.

2.3- Adaptability

A self-organising system must be robust against perturbations and therefore adapt itself to changes. If not, there would be only single points in the parameter space where self-organisation could happen and it would vanish the moment any parameter would suffer the smallest change.

Noise, or non-deterministic fluctuations, happen in every physical system. Therefore, embodied systems will experience it when interacting with the environment. Self-organised systems must then be tolerant and adapt to noise fluctuations (see [Heylighen, 2001] for a review of early work on self-organisation, where the importance of noise is duly emphasised). This means that basins of attraction of global behaviour attractors can not be too narrow, otherwise the system will easily escape the attractor under small perturbations.

On the other hand, noise is quite a positive feature for self-organisation in general (also suggested in [Parrish and Edelstein-Keshet, 1999]). In [Helbing and Vicsek, 1999] small noise fluctuations are considered responsible for allowing self-organising systems to escape local optima and achieve optimal self-organised behaviour.

This brings us to another feature of self-organisation, necessary for adaptability: negative feedback. Systems subject to noise fluctuations, which includes all embodied systems, can not do without it. Otherwise they become unstable.

Negative feedback has been associated with hard limits, such as exhaustion of resources or population members [Bonabeau et al., 1999, Heylighen, 2001]. While this may be the case in some systems, it can hardly be generalised. Ant recruitment limits, for instance, are not always due to deployment of all ants in the colony—in [Cassill, 2003], for various concentrations of food sources, the nest always maintains a significant (though varying) percentage of unoccupied ants. This means that, in this case, there is an intrinsic feedback mechanism far from the hard physical limit.

2.4- Interaction

A self-organised system is necessarily composed of several elements. The resulting behaviour of the system must entail a correlation of actions to produce an organised behavior, under some criterion. Without interaction, components would just be a bunch of independent entities unable of any coherent behaviour.

Interaction may take place directly between elements or through the environment [Anderson, 2002]. In the latter case we have *stigmergy*, a term that was coined by Gassé precisely to describe interaction through the environment [Bonabeau et al., 1999].

Interaction may be *cooperative* or *competitive*. Competitive interaction happens whenever self-organisation is needed to share a resource among system components. In situations such as fluid convection [Normand et al., 1977] and pedestrian traffic [Helbing and Molnar, 1997] there is a competitive interaction. In particular, elements going in opposite directions in a section of limited width or area, self-organise into flows or lanes.

In cases such as ant trails [Bonabeau et al., 1999] or article moderation on the slashdot web site [Johnson, 2001][1] there is a cooperative interaction involved. Ants cooperate in fetching food to feed the colony and slashdot users cooperate to evaluate and publish posted articles. Cooperative interaction happens when system components contribute to perform collectively one single task.

2.5- Asynchronism

Asynchronism means that there is no form of global synchronisation. In any physical system, a signal takes time to propagate. Therefore, even external cues will not be perceived at exactly the same time by all components of the self-organised system.

Besides signal amplitude fluctuations, timing fluctuations are also a form of noise. They take the form of delays. All these noise forms are part of the self-organised system and of its interaction with the environment. Synchronous discrete behaviour does not exist in embodied systems. It is an abstraction based in instantaneous signal propagation, that is only implemented in computers.

Usually, synchronisation signals are provided by external sources, which is excluded by the first fundamental property (in 2.1). However, it is important to name asynchronism as a property because systems with internally generated “perfect” synchronisation signals must also not be considered self-organised. Instantaneous propagation is not possible in any way.

Asynchronism implies that each component may perceive and react to a signal with timings that may be slightly different from other components. This variability is a result of noise fluctuations and absence of instantaneous signal propagation.

2.6- Comments on the properties

The first three properties are consensual in the literature [Kauffman, 2005, Shalizi, 2001, Anderson, 2002, De Wolf and Holvoet, 2005]. The fourth is mentioned in [Bonabeau et al., 1999] together with: *Positive feedback*; *Negative feedback*; and

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[1] - http://slashdot.org/faq/com-mod.shtml#cm520

Fluctuation amplification. Negative feedback is already considered in adaptability. Fluctuation amplification is a result of positive feedback and this is not a general property of self-organisation (see discussion and examples in [Anderson, 2002]).

The need for the last fundamental property, asynchronism, is supported in research that goes back to 1983. The problem of discrepancies of discrete simulations was raised in [Choi and Huberman, 1983]. In 1984 it is found that asynchronous one-dimensional Wolfram Cellular Automata does not have the same behaviour as their synchronous versions, displaying less structure [Ingerson and Buvel, 1984]. More recently, the simulation of social games based on grids with synchronism also showed to suffer from severe unrealistic results [Huberman and Glance, 1993].

2.7- The exclusion of Cellular Automata

We will now discuss the particular case of Cellular Automata (CA) due to their widespread use. They are not self-organised, according to the fundamental property in . Common use of CA updates cells synchronously, in parallel. This means that there is a global external synchronisation signal, which controls instantaneously the behaviour of the system. At that signal all cells are updated simultaneously. This clearly violates the asynchronism property.

Research has plentifully confirmed collapse of behaviour by simply operating CA in asynchronous mode. A generalized freezing in the Game of Life is shown, when cells are updated asynchronously [Bersini and Detours, 1994]. This would be the more realistic situation for simulating embodied systems, since it requires no global synchronisation signal. In this updating policy, a different cell is chosen at random for updating, until all *LxL* cells have been updated (*L* being the number of cells on each side of the grid). In the same work, a similar radical change of behaviour towards stability was also found in Hopfield networks. In normal operation, these networks are supposed to be asynchronously updated [Hopfield, 1982]. Authors of [Bersini and Detours, 1994] consider that this result holds for all members of Wolfram's class IV CA, making them lose their rich characteristics.

Therefore we argue that CA updated in parallel (synchronously) are not self-organising systems, independently of any observable increase in complexity or in order. An asynchronous updating procedure will provide compliance with the asynchronism property. To be self-organised it will still have to verify the other four properties.

3- Consequences of self-organisation properties

In this section we analyse specific aspects found in the behaviour of self-organising systems, relating them to the fundamental properties previously presented. We are particularly interested in systems both self-organised and with emergent properties. This includes all natural physical and chemical systems.

3.1- Minimisation of conflicts

The more complex the system becomes the more difficult it may be to discern between which form of interaction is relevant, since the two may show simultaneously, but at different levels. In [Dussutour et al., 2004] observations show that ants cooperate bringing food to the nest, but in narrow passages they compete for the right to pass and sometimes push others backwards. The overall result, in this case, is still a cooperative interaction, despite that some local interactions are competitive.

In reality even when there is competitive interaction there must be some sort of cooperation between individuals. As is pointed out in [Parrish and Edelstein-Keshet, 1999] regarding animal aggregation, with maximal short-term selfishness in the components, global behaviour is compromised. Deadlocks may occur and the system does not stand as a whole. On the other hand, a local behaviour tuned for maximum group benefit is not viable either. In many cases it would require global knowledge at a local level, which is not possible. An altruistic behaviour would also violate the principles of evolution. A balance is needed somewhere between these two extremes.

It turns out that the problem is a general one of self-organisation and not limited to animal aggregation. The solution to the conundrum can be found in [Helbing and Vicsek, 1999]. It is shown that global efficiency of opposite pedestrian traffic is maximised when interaction rate is minimised for each component. When this happens two separate lanes form, one in each direction. The minimisation of interactions follows directly from maximising the average velocity in the desired direction. The same result applies to particles of identical size and different density moving in a column of viscous fluid (light particles rising and heavy particles sinking). Results were modeled but are coherent with observed empirical data. The importance of this relation is that a local performance maximisation results also in a global performance maximisation, in spite of possible local antagonistic interactions. In [Heylighen, 2006] minimisation of conflict is hinted at, and considered as dynamic self-coordination through a medium.

The generality of this result is reinforced by similar findings on the origin of RNA secondary structure development [Schultes et al., 1999] and in the problem of island formation in epitaxial growth on materials technology [Gonçalves and Mendes, 2002]. In the former case, RNA secondary structure self-organises in a way to minimise energy of conflicting intramolecular interactions. In the latter, when depositing a particle in a substrate, the particle minimises repulsive energy towards previously deposited particles and the resulting self-organisation takes the form of *islands*.

The local rule of minimisation of interaction intensity is consistent with the selfishness necessary for an evolutionarily sustainable strategy. The fact that this local rule results in a maximal global efficiency of the self-organised system also favours the global system to have a selective advantage. This establishes a solid basis for natural selection to have evolved self-organisation in different forms. In cooperative interaction there is a natural advantage resulting from the self-organised system. In competitive interaction the minimisation of energy spent in conflicts is maximally efficient to the global system.

3.2- Symmetry and symmetry breaking

Symmetry of local rules is an important aspect of self-organisation. Systems with both self-organisation and emergent properties are constituted by seemingly identical elements, with identical properties and identical individual behaviour. Therefore, by specification, their interactions are locally symmetric. In identical conditions all components act in the same way towards each other and towards the environment.

In systems of driven entities, interaction symmetry is considered to provide optimal self-organization [Helbing and Vicsek, 1999]. Optimal in the sense that there is a function, expressing energy for instance, that is minimised with such interactions. However, a break in symmetry is important to allow self-organisation. Among other cases, we mention Ethernet MAC, where each station will back-off for a random amount of time after a collision, thus potentially breaking the simultaneous interaction that generated it [Fukuda et al., 2000].

Notice that in embodied systems, symmetry breaking can simply be achieved by the effect of noise fluctuations. There is no need for explicit local rules for symmetry breaking if the elements are subject to noise fluctuations in their interactions. Symmetry naturally breaks in that case. This idea was first advanced apparently by Turing [Turing, 1952]. Its importance to the origin of life is discussed in [Hartman, 2000]. Another interesting case of noise fluctuations inducing symmetry breaking is genetic drift in

Genetic Algorithms. It happens only due to the non-deterministic character of selection, which induces fluctuations. At some point one of the fluctuations may become important enough to generate a drift in the population.

3.3- Variations in complexity

A mere increase in complexity can be observed in all self-organised systems, from physical to social. However, we suggest that living systems display a sequence of increase and decrease in complexity without external variation of parameters.

Let us consider the case of ants. Food transport is done via a trail, which is an organised behaviour with a certain complexity. Nevertheless, a small percentage of ants keeps exploring the surroundings and if a new food source is discovered a new trail is established, thereby dividing the workers by the trails [Hubbell et al., 1980] and increasing complexity. Factors like depletion of food sources [Bernstein, 1975] or reduction of the number of workers in a nest [Beekman et al., 2001] will entail a disorganised behaviour, hence a reduction in complexity. Even a phase transition, with hysteresis, is found in the food gathering behaviour of ants, as a function of the colony's size [Beekman et al., 2001].

On the other hand, physical systems present many instances of self-organisation under the classification of convective phenomena. Maybe the most widespread among studies in self-organisation are the Bénard cellular patterns, but other convection examples can be found in atmosphere and oceans and in evaporation. This type of self-organisation takes place in highly inhomogeneous media or when there is an interaction between some transport phenomenon and external constraints, such as gravitational, electrical and magnetic fields [Normand et al., 1977].

We may clearly observe the difference between these cases. Physical and chemical systems are completely dependent on the external constraints or cues to exhibit self-organisation. Once the cue assumes values in a relevant interval, the system will display self-organisation, which will cease when the cue gets out of the interval.

In biological systems, there is no such direct correspondence. External cues usually play a role of triggering some sort of self-organisation, such as aggregation in animals. After the behaviour is triggered the whole group will continue with its own dynamics even after cue withdrawal. As examples of similar behaviour we find tuna fish schools [Parrish and Edelstein-Keshet, 1999] and ant nest migration [Smallwood and Culver, 1979]. These internal dynamics include periods in which complexity decreases. After migrating the nest, the colony will look for food sources in less complex formations (seemingly random exploration) and then reorganise again in trails, etc. As another example, bees stay in the nest for long periods [Moore, 2001], which is clearly a less complex arrangement than when they are exploiting a food source.

This leads us to hypothesise that in living organisms self-organisation will manifest itself by successive periods of increasing complexity, followed by others of decreasing complexity, with minimal or no external influence. This autonomous dynamics in the variation of complexity and, therefore, of organisation, is typical of living systems and completely absent in the non-living ones.

4- Conclusion

Self-organised systems behaviour results much from their insertion in an environment they interact with. The definition of the five fundamental properties of self-organisation took that into account. Among them, the property of asynchronism is newly introduced to help clarify the definition of self-organisation. As a result it was shown why common Cellular Automata are not self-organised.

We have also analysed some behaviour aspects of self-organisation, namely minimisation of conflicts, symmetry breaking and complexity variations. This set of specific aspects may help us to design systems, at the local level, taking into account what they will show at the global level.

This research also suggests some problems to explore. For instance, evaluation of self-organisation in CA with asynchronous updating rules requires more research. Further data is necessary to verify the hypothesis of distinguishing living from non-living self-organised forms, by observation of complexity variation. Other cases should also be studied, to confirm optimisation of global behaviour from minimisation of local competing interactions.

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