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RESEARCH

A Novel Shortcut to Cubical Perspective Drawing

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Abstract
The article introduces a novel construction that simplifies the drawing of cubical perspectives. A recent work described a method for drawing cubical perspectives which classifies lines into several types and renders each type systematically. Some of these types require an auxiliary external construction, hindering the drawing process. This research proposes a novel method that shortcuts this step with an internal construction. This method both accelerates and simplifies the studio process, reduces systematic errors, and makes freehand sketching more viable and precise. Next, the shortcut is applied through the case of the Solimene factory (Italy), an organic architecture that highlights both the issues inherent to cubical representation and the advantages of the proposed shortcut.

Keywords Cubical perspective · Spherical perspective · Handmade drawing · Immersive drawing · Solimene factory

Introduction

Perspective drawing allows one to preview architecture before it is built. It aims at *mimesis*: the illusion of seeing the imagined object, which spherical perspective takes further through *immersion*. A spherical perspective is a flat drawing that

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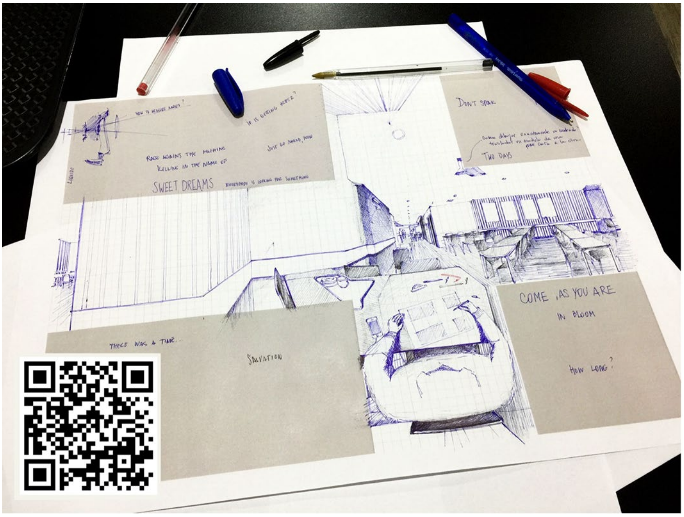

Fig. 1 Urban sketch in cubical perspective and its VR view (QR). Lisbon 2018 (All images are by L. F. Olivero, unless specified otherwise)

contains all the visual information that can be seen from a given point (Fig. 1), hence it can be turned into a VR visualization. Research into spherical perspectives has recently been thriving due to its connection with 360° photography and VR panoramas. Several perspectives, including equirectangular (Araújo 2018b), azimuthal equidistant (Araújo 2018a), Mollweide (Santoyo and Santoyo 2021) and cubical (Araújo et al. 2020) have shown important theoretical developments in recent years. In particular, Araújo et al. (2020) provided a full method for drawing a cubical perspective. However, this method is hindered by a special case of line rendering that requires an awkward auxiliary construction. In response, the present paper has two goals: first, to show that the complex construction of Araújo et al. (2020) can be replaced by a simpler one that does not require external auxiliary diagrams. Second, to provide a case study to test applied cubical perspective within a real and complex architectural setting. The paper introduces the original construction of Araújo et al. (2020), then explores the new construction, its mathematical proof, and finally tests the new construction's application through a case study.

Historical Introduction

The history of immersive perspective starts with conical projections onto partially immersive surfaces, such as curved or irregular ceilings (Pozzo 1693; Bortot 2015), apses (Rossi et al. 2018), perspective boxes (Koslow 1967; Fukuoka 2005; Verweij 2010; Spencer 2018) and, in the nineteenth century onto cylindrical *panoramas* (Bordini 1984; Grau 2003). These representations were obtained via re-projected and traced preliminary linear perspectives onto the final surface through *ad-hoc* physical mechanisms (grids, lights, needles) (Pozzo 1693: 220–221; Grau 2003: 52;

Candito 2010: 53). The cylindrical case is interesting as it is easy to build, isometric to the plane, and always projects onto sinusoidal lines in a constant curvature surface that facilitates uniform lighting. Still, it is not fully immersive.

The sphere is the natural fully immersive surface, but it is non-developable, so one cannot get the spherical drawing by directly stitching linear perspectives. The cube is a nice compromise as it is fully immersive and simply joins six linear perspectives. However, its corners create physical cues that break immersion (Grau 2003: 13–14). These difficulties disappear with digital media where virtual viewer placement and lighting are fully controllable, making both sphere and cube more credible and viable fully immersive environments (Jerald 2015: 15).

Since its introduction in the '80s, *cubical environmental mapping* has seen continued applications (Cruz-Neira et al. 1992; Israel et al. 2009; Boddien et al. 2017) with different and sometimes better features than the sphere (Greene 1986). However, research into this field (Grimm and Niebruegge 2007; Wong et al. 2007; Lambers 2019) focused on *computer rendering*; cubical *drawing* as a research problem in *technical drawing* only resurfaced recently (Olivero and Sucurado 2019; Olivero et al. 2019; Araújo et al. 2020).

The technical drawing (e.g., descriptive geometry) of a cubical perspective presents particular geometrical problems. The main challenge consists of the fragmentation of lines since a general line projects on up to four segments in four cube faces. When seen on the flattened cube –where one wishes to draw– the segments will meet at variable angles and sometimes on disjoint faces (Araújo et al. 2020: 3) (Fig. 2).

Historically, line fragmentation was seen in projections onto connected planes such as walls, ceilings (Nicéron 1638; Bosse 1653) or inside perspective boxes. Nevertheless, they presented a related yet different problem. For example, perspective boxes were *parallelepipedal*, not cubical, and its content was created from a non-central viewpoint onto the closed, not flattened box (Verweij 2010: 47; Gay and Cazzaro 2018: 9; Spencer 2018: 4). The solution considered the 'Euclidean theorem that if two straight lines meet at an angle, they appear to be [colinear] if viewed on the same level' (Grau 2003: 52). In practice, artists pinpricked references within the closed box with a needle coming from the peephole and passing through a classical perspective (Grau 2003: 52). These *ad-hoc* solutions are inappropriate for cubical perspective which requires a general method for drawing on the flattened cube. This issue was first seen as a problem of angles (Olivero et al. 2019) and later solved by considering cubical drawing as a case of spherical perspective (Araújo et al. 2020).

Spherical and Cubical Perspective

The theoretical development of spherical perspective was hindered by two misconceptions: the supposed contradiction between Euclidean Optics and Linear Perspective (Brownson 1981), and the artificial distinction between 'real perspective' and anamorphosis, the latter being considered a mere game or perversion of perspective (Migliari 2005: 34; Cabezos Bernal et al. 2014: 137). A recent work

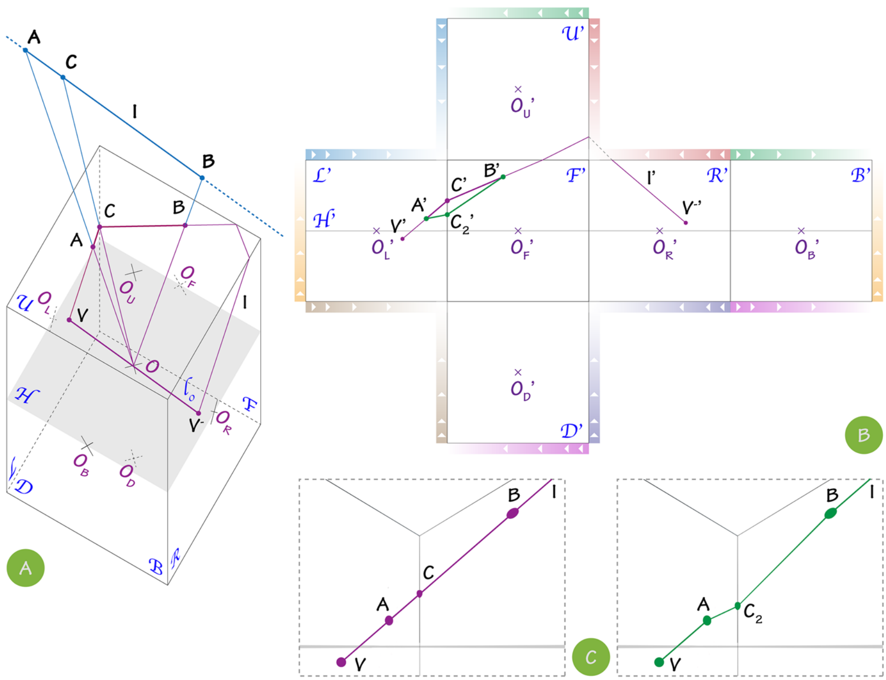

Fig. 2 Line fragmentation problem (A, B): only point C' makes the line l look straight in VR (C)

reconciles Euclidean Optics with Linear Perspective by reframing anamorphosis as the fundamental mathematical object from which perspective derives (Araújo 2021a). In this framework (hereon used), anamorphosis is an equivalence relation between 3D objects which generalizes Euclid's optics and is independent of the projection surface. This allows a canonical definition of vanishing points *before* perspective, the latter defined as a two-step process: a spherical anamorphosis (mimetic and unique) followed by a *flattening* (neither unique nor mimetic) onto a bounded region of the plane (Araújo 2018a: 148). Perspective is to anamorphosis what a world map is to the globe, its purpose is to store visual information conveniently on the flat surface and, like in cartography, no flattening is perfect – each being merely convenient for some purposes. This approach, validated by its useful relation with VR visualizations, discards the older goal of finding a 'natural perspective' with minimal distortions (Barre and Flocon 1967).

This framework switches focus from lines to spherical geodesics where each spatial line projects on the sphere as a half geodesic (a meridian), always ending at two vanishing points, diametrically opposite (antipodal) on the sphere (Masetti 2014: 127; Araújo 2021b: 9–12). Each spherical perspective requires different drawing methods, but this framework provides a strategy for solving each case, based on a classification of geodesics and consideration of the symmetries of the flattening. This strategy was validated by its successful use, first in solving the azimuthal equidistant perspective (Araújo 2018a) (generalizing the 'fisheye' perspective of Barre and Flocon to the full sphere (Barre and Flocon 1967)), then

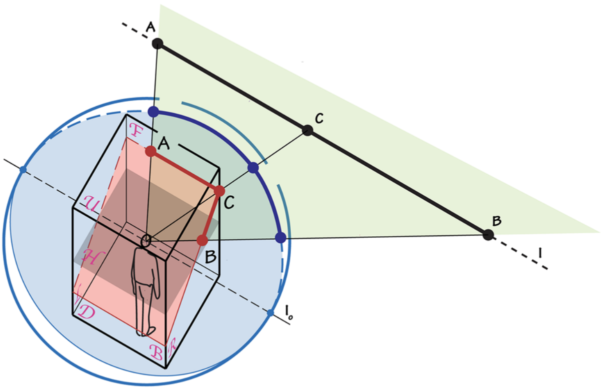

Fig. 3 A segment projecting onto a geodesic arc and its cubical anamorph

the equirectangular (Araújo 2018b), and finally the cubical perspective (Araújo et al. 2020).

The cubical case was solved by noticing that a cube and a concentric sphere are radially equivalent (Fig. 3). Thus, cubical perspective was treated as a special case of spherical perspective where the flattening contains a sub-step of projection from sphere to cube. The fragmentation problem was then handled as the classification of spherical geodesics on the cube rather than an *ad-hoc* joining of six linear perspectives. Thus was obtained a complete solution with a classification of geodesics and a prescription of rendering methods using descriptive geometry for all geodesic arcs between any given pair of points on the cube (Araújo et al. 2020).

Advantages and Disadvantages

Cubical-spherical perspective is attractive because each individual face has the familiarity of linear perspective, while the total cube provides the full immersion of spherical perspective. Relative to the 'curvilinear' spherical perspectives, it has the advantage of using exact constructions for line renderings, while the equirectangular and fisheye cases require approximations. In cubical perspective one can use techniques from linear perspective (e.g. for multiplication of regular elements) but also enhance and generalize these through the fact that all lines now have two vanishing points available inside the picture plane. The connection with linear perspective simplifies teaching, as it can build on the regular courses given to architects and designers. A beginner can start by drawing 'normally' on each face and gradually acquire further techniques, something important since curvilinear perspectives are scarcely included in the educational curricula (Kulcke 2019: 345).

The limitations of cubical perspective are its large casuistry of geodesics and rendering methods (an obstacle for teaching), the discontinuities of the cubical map (which complicate drawing across edges), and the lack of the symmetry groups of the equirectangular and fisheye cases. Another problem consists of converting the drawings into VR environments, which is theoretically trivial (Grimm and

Niebruegge 2007; Dimitrijević et al. 2016) but is an awkward multi-step process in practice. This latter problem was addressed by the Spheri platform (Olivero and Araújo 2022; Olivero 2023), which converts spherical perspectives into VR on-the-fly. This article focuses on the first problem by providing a method that streamlines a whole class of drawing cases, hence simplifying the use of cubical perspective and stimulating its application.

Notation and Problem

This section establishes the notation and constructions to be used throughout the whole article. For a full definition of cube flattening and cubical perspective image consult Araújo et al. (2020).

Denote elements in space as $\boldsymbol{P}$ (bold lettering), their conical projection onto the cube's surface as P (regular lettering), and their cubical perspective image within the cubical map as P' (prime index). Denote the centre of the cube by the letter $\boldsymbol{O}$ and call horizon (denoted $\boldsymbol{H}$) to the plane $\boldsymbol{OO_FO_R}$ (Fig. 2). Call horizontal (resp. vertical) any plane parallel (resp. orthogonal) to $\boldsymbol{H}$. Call height of $\boldsymbol{P}$ (denoted $z(\boldsymbol{P})$) to the distance of $\boldsymbol{P}$ to $\boldsymbol{H}$. Given a point $\boldsymbol{P}$ we say its antipode is the point $\boldsymbol{P}^-$ such that $\overrightarrow{\boldsymbol{OP}} = -\overrightarrow{\boldsymbol{OP}^-}$.

Two points in space $\boldsymbol{A}, \boldsymbol{B}$ define a line $\boldsymbol{l} = \boldsymbol{AB}$ and a plane through $\boldsymbol{O}$. The intersection of this plane with the sphere centred in $\boldsymbol{O}$ is a geodesic. Geodesics are essential to plot lines: the conical projection of $\boldsymbol{l}$ on the sphere is half of that geodesic, ending at two antipodal vanishing points. Call *geodesic* to both the intersection of that plane with the concentric cube and its image on the flat cubical map. The main problem of cubical perspective is to draw the geodesic that passes through two points whose images are given on the cube. The most laborious case arises when the projections A, B fall on two adjacent faces (Olivero et al. 2019; Araújo et al. 2020) and we need a third point C on the edge between them to get the cubical perspective image of segment $\overline{\boldsymbol{AB}}$.

Construction (Fig. 4)**:** let $\boldsymbol{A}, \boldsymbol{B}$ be two points in space. Let g be the conical projection of the geodesic of $\boldsymbol{l} = \boldsymbol{AB}$. Thus, $A, B \in g$. Suppose that $A \in F_A$ and $B \in F_B$ such that F_A and F_B are adjacent faces of the cube. Let e be the edge between F_A and F_B and l_e the vertical line through e. Let $\boldsymbol{\delta_e}$ be the plane through l_e and $\boldsymbol{O}$. Let $\boldsymbol{C} = \overline{AB} \cap \boldsymbol{\delta_e}$. Then, $C = \overline{\boldsymbol{OC}} \cap l_e$.

The image of $\overline{\boldsymbol{AB}}$ onto the cube's surface is the union of $\overline{AC}$ and $\overline{CB}$, and therefore the image of $\overline{\boldsymbol{AB}}$ in the cubical map is $\overline{A'C'} \cup \overline{C'B'}$ (Fig. 4B). The question, then, is how to get point C' within the flat cubical map using descriptive geometry. The solution proposed by Araújo et al. (2020: 39) for finding C' follows Fig. 5: project $\overline{AB} \cap \boldsymbol{\delta_e}$ onto an auxiliary plane perpendicular to $\boldsymbol{\delta_e}$ to find C_ε. In the cubical map, C_ε' determines C_{F_B}', the cubical perspective image of the orthogonal projection of $\boldsymbol{C}$ on face F_B. Then $O_{F_B}'C_{F_B}'$, the cubical perspective of the orthogonal projection of $\boldsymbol{OC}$ onto F_B, determines C' on l_e'. We will refer to this as the Old Construction from now on.

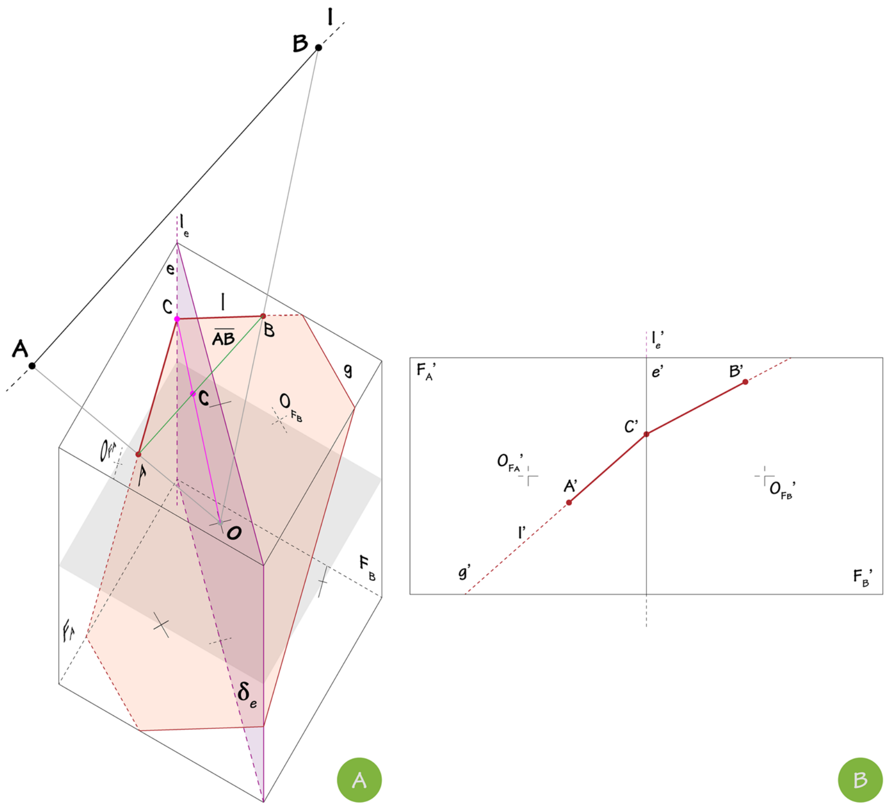

Fig. 4 Construction in space (**A**) and in the cubical map (**B**)

Problem: Can a different construction of C' be found that requires no external auxiliary plane?

The New A-Construction

In contrast to the Old Construction, the new A-Construction gets point C' within faces $F_A{}'$ and $F_B{}'$ right after A' and B' are plotted (Fig. 6B). All what we need for getting C' is the same spatial elements stated in Fig. 4 but without the vertical plane $\boldsymbol{\delta_e}$ (Fig. 6A).

A-Construction (Fig. 7)**:** Let $A', B', l_e{}'$ be the image of A, B, l_e in the cubical map. Suppose $z(A') \neq z(B')$. Let $A_e{}'$ be the foot of the perpendicular from A' to $l_e{}'$. Let $X_e{}' = \overline{A'B'} \cap l_e{}'$. Let $h_{Xe}{}'$ be the horizontal line passing through $X_e{}'$. Let $X_{F_B}{}' = h_{Xe}{}' \cap \overline{A_e{}'B'}$. Let $O_{F_B}{}'$ be the centre of face $F_B{}'$. Let $X' = O_{F_B}{}'X_{F_B}{}' \cap l_e{}'$.

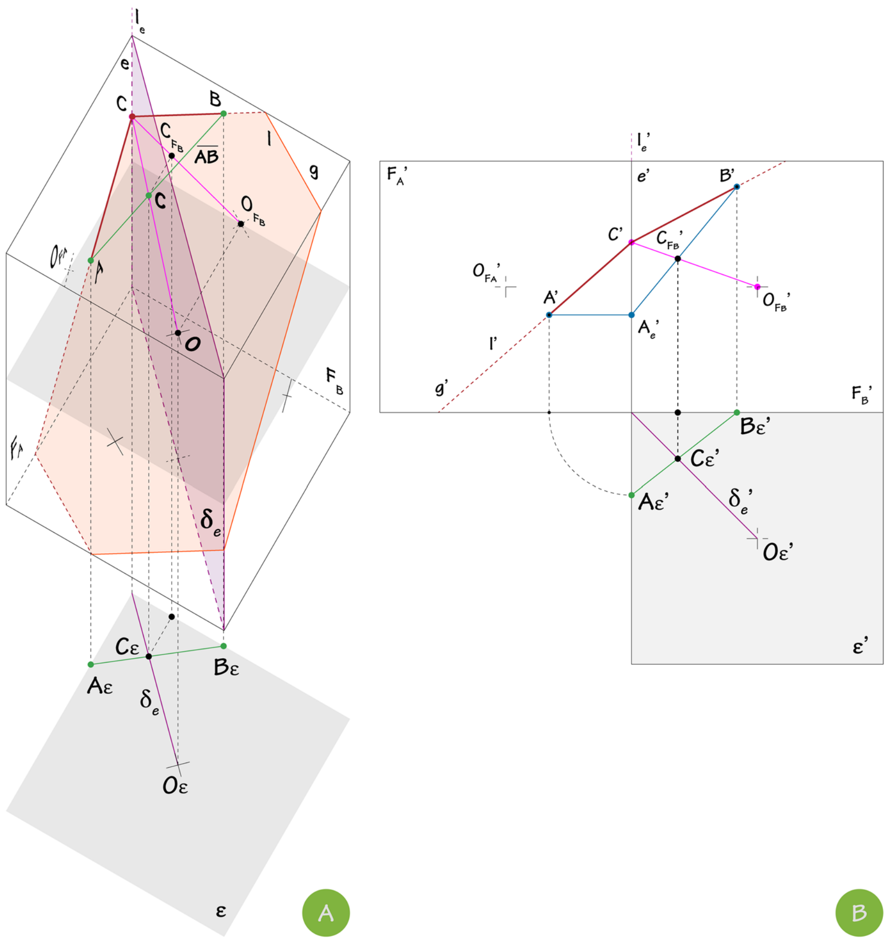

Fig. 5 Old Construction in space (**A**) and in the cubical map (**B**)

Theorem: $X' \in g'$*and the image of* $\overline{\boldsymbol{AB}}$ *on the cubical map is* $\overline{A'X'} \cup \overline{X'B'}$.

The theorem follows by relating the two constructions:

Theorem: $X'(A - Construction) = C'(Old\ Construction)$

which in turn follows trivially from the following lemma, which we will prove:

Lemma: $X_{F_B}{}'(A - Construction) = C_{F_B}{}'(Old\ Construction)$.

Proof (Fig. 8)**:** Consider the construction of Fig. 4, where A and B are the points on the cube and $\boldsymbol{C}$ is the intersection of $\overline{AB}$ with the vertical plane $\boldsymbol{\delta_e}$. Let $\boldsymbol{\beta}$ be a vertical plane not intersecting the cube that makes an angle of $\alpha = 45^\circ$ with respect to both

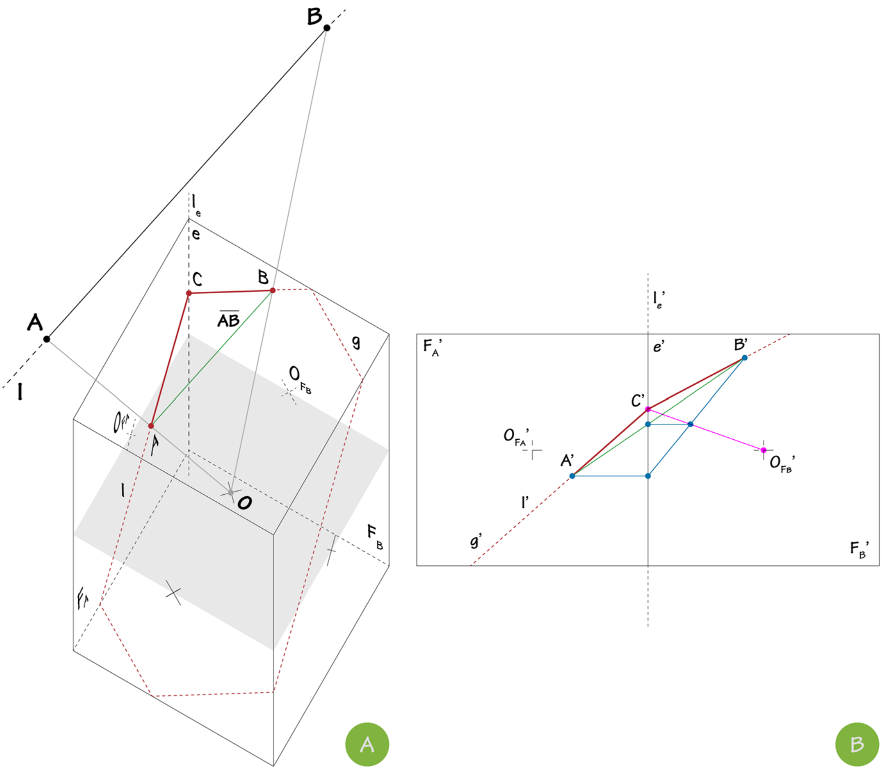

Fig. 6 Spatial elements needed for the A-Construction (**A**). A-Construction in the cubical map (**B**)

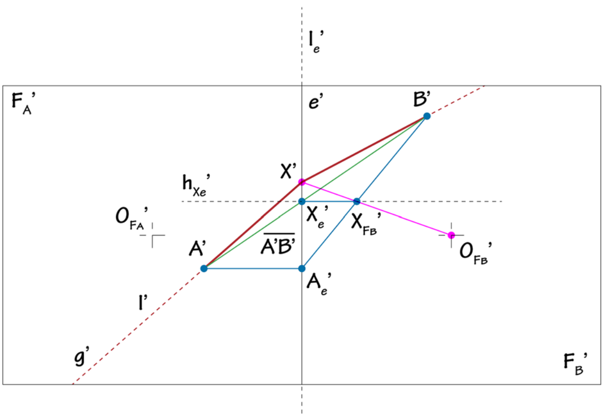

Fig. 7 A-Construction in the cubical map

F_A and F_B (Fig. 8A–D). Let f be the orthogonal projection onto $\boldsymbol{\beta}$. Use the notation $P'' = f(P)$ (double prime) to denote images of points by f.

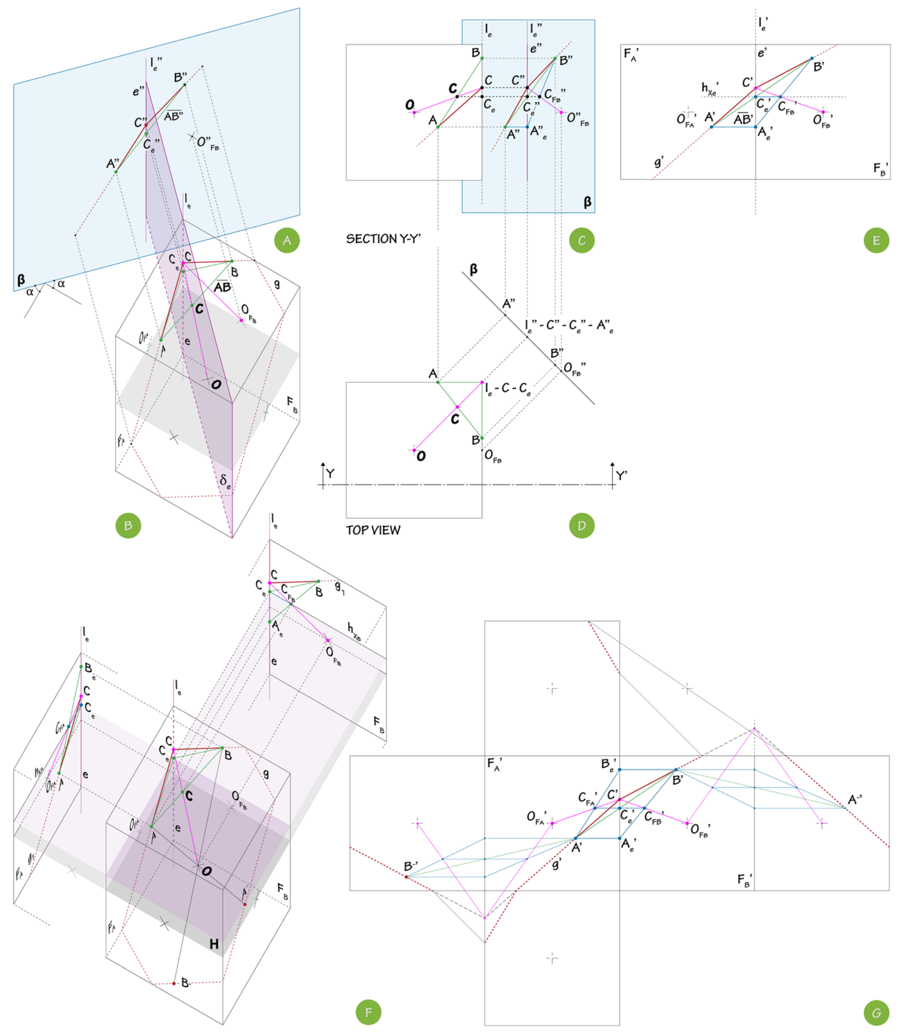

Fig. 8 A-Construction, projections on plane $\boldsymbol{\beta}$ (**A**-**D**), A-Construction with antipodes (**F**, **G**)

We say that three points are in the relation $A - B - C$ if they are in the same line and B is between A and C. Because f is an orthogonal projection it preserves the collinearity of $A - \boldsymbol{C} - B$, that is, we have $A'' - \boldsymbol{C}'' - B''$. Let C_e be the point at l_e that has the same height as $\boldsymbol{C}$ (Fig. 8C). Because $\boldsymbol{\beta}$ is a vertical plane f preserves the heights of the points. Then $A'' - C_e'' - B''$.

$\overline{A''C_e''}$ (respectively $\overline{C_e''B''}$) relate to $\overline{AC_e}$ (respectively $\overline{C_eB}$) by a horizontal scaling around l_e. In fact, because $\boldsymbol{\beta}$ makes an angle of 45° with both F_A and F_B (Fig. 8D), we have $d(A'', l_e'') = d(A, l_e)\cos(45°)$ and $d(l_e'', B'') = d(l_e, B)\cos(45°)$. Since horizontal distances are scaled on both

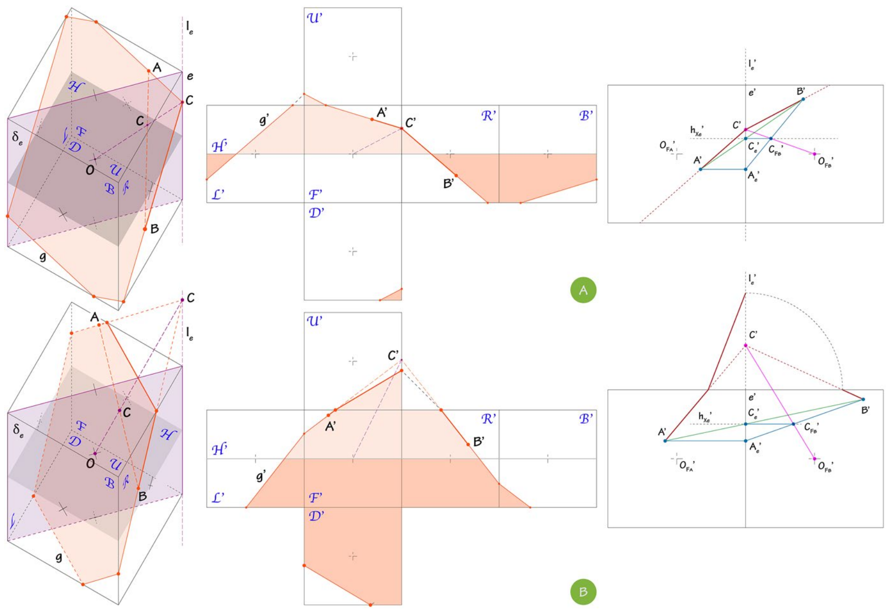

Fig. 9 Application with the Auxiliary point C' inside e' (**A**) and outside (**B**)

sides of l_e by the same factor while vertical heights are preserved, the slopes of AC_e and C_eB remain equal to each other on both sides of l_e.

The perspective images A' and B' are obtained from A, B by a rotation around the folding line l_e, so the perspective image preserves both the heights and the horizontal distances from l_e to each of these points, and furthermore preserves the position of $C_e \equiv C_e{}'$. Hence the slopes are again preserved, and therefore $A' - C_e{}' - B'$ (Fig. 8E). Hence $C_e{}'$ is the intersection of $A'B'$ with $l_e{}'$, that is, $C_e{}'$ equals the $X_e{}'$ defined in the A-Construction, and therefore we have shown that $X_e{}'$ has the height of $\boldsymbol{C}$ (compare Figs. 5B and 7). Then $X_{F_B}{}'$, which is the point of $A_e{}'B'$ in the same horizontal as $X_e{}'$ also has the height of $\boldsymbol{C}$ and therefore point $X_{F_B}{}'$ of the A-Construction equals the $C_{F_B}{}'$ of the Old Construction. **Q.E.D.**

Both theorems above follow as corollaries, so the cubical perspective image of $\overline{\boldsymbol{AB}}$ is $\overline{A'X'} \cup \overline{X'B'}$, where X' is obtained by the A-Construction.

This shortcut covers all the most difficult cases with C' within or outside e' (Fig. 9)[1]. Special cases: (1) the A-Construction assumes $z(A') \neq z(B')$ otherwise $X_{F_B}{}'$ is undefined. If $z(A') = z(B')$, take either antipode (say $\boldsymbol{A}^-$) and then $z(A^{-\prime}) \neq z(B')$ and the construction can be applied, as antipodes fall on the opposite faces but on the same geodesic (Fig. 8F–G). (2) Adjacent cube faces may be non-adjacent on the cubical map. This also happened in the Old Construction, and

1 Details in (Araújo et al. 2020: 41).

the solution is the same as in that case: face images are made adjacent through translation and rotation/reflection, or antipodes are used instead.

Case Study – Drawing the Solimene Factory

Hybrid Immersive Models (HIM)

The term ‘modelling’, applied to all possible 2D or 3D replicas, derives from the manifest appearance of objects. A model’s degree of iconicity is related to the gap between the real and the imaginary (Moles and Rohmer-Moles 1980)[2], a gap exploited by critical analysis, design, and graphic art.

In the field of representation, a sketch expresses the designer’s hand and their ‘flashes of the intellect’ (Scolari 1982), capturing the essence of the concepts manipulated by the author. With the use of a computer, the sketch can also be scanned, and the image captured by a bitmap can be transformed into a 3D information model (Migliari 2003). A third element is interposed between hand and mind as programmed instructions guide choices within pre-constituted possibilities. In the presence of software mediation, the technical skills and intentions of the operator are guided, leading one to wonder what is left of the speed of execution, intuition and immediate measurability of freehand drawing.

This question was addressed within the *Advanced Representation Techniques* course[3]. The year’s theme comprehended the development of digital landscapes generated from immersive drawings following the cubical-spherical perspective and explored through VR/AR/MR. Cubical perspective provided an opportunity to discuss the advantages/disadvantages/potential of classical and spherical perspectives, and hybrid immersive model (HIM) (Olivero 2021).

A HIM enables the scholar to make perceptible the selective processes behind the sketch and the speculative (not only spectacular) visualization of its interactive multimedia space. This 3D fruition recovers a psycho-physical experience in which users interact and dialogue with the selection of elements filtered by the designer’s eye and mind.

Another difficulty arose with the designers’ professional profile, which requires the creative, technical and communicative functions related to product creation (Product Designer), and the content’s environment and meaning (Interior Designer).

The challenge, addressed both at a didactic level and as applied research, was carried out within ‘technical’ representations, where autographic possibilities are notoriously more limited than in the allographic field. The organisation of abstract

[2] Moles identifies 10 levels of iconicity.

[3] Advanced Representation Techniques—Product Design (TAR) course, Professor A. Rossi, academical years 2018/19 and 2019/20, master’s degree Design for Innovation, Engineering Department, University of Campania, Italy. Instructor: L. F. Olivero (PhD, Unicampania 2017/21, supervisor A. Rossi, international supervisor A. B. Araújo). Group 1: A. D’alessio, T. Di Palma, C. Crispino, M. Campanile, M. Petrillo, L. Villani; Group 2: I. Jayed, A. Ambrosio, M. Di Fuccia, A. Perrotta, S. Carleo, M. Petrosino.

and figurative elements in the composition had to conform to codes and canons, but not to avoid the search for meaning. This is an ongoing issue for scholars of Representation, to clarify the critical value revealed in the surveyed spaces, or conversely, the sequence of symbolically growing representations in the imagined space.

In this panorama of studies and research, it is easy to find literature discussing the instruments' metric reliability or methods' accuracy. Nevertheless, it is unusual to find studies identifying the heuristic peculiarities of workflows to promote a dialogue between tradition and innovation. Geometric logic is useful for this purpose, a tool forcing the seeming appearance of shapes, comparing readings and interpretations on the basis of a method useful for distinguishing what initially appeared 'smooth' and indistinct (*laevo*) from what instead turns out to be fundamental for analysis and synthesis (*re-laevo*) (de Rubertis and Masiero 1995).

The Solimene Factory

The Solimene complex, built between 1952–55 in Vietri sul Mare (Campania, Italy) by Paolo Soleri, proved a stimulating opportunity for our purposes. The building was surveyed with Total Station and direct verification (Rossi 2017), Terrestrial Laser Scanner and photographic datasets (Rossi and Palmieri 2020), photogrammetry and Structure from Motion (SfM) techniques, modelled with indirect and direct methods (Rossi and Palmieri 2022), completed with parametric models (Rossi et al. 2024b) and combined in an accessible information system aimed for restoration (Rossi et al. 2024a).

Here, the procedure integrates analogical and digital modalities (which have hitherto been distinct and frequently opposed), using cubical perspective to select and filter the architecture, and automated procedures to correct and validate the creation of spherical panoramas thus converted into the cubical format. This procedure communicates the configuration space's reading in an immersive way, where the relationships between elements renews contents and communication to detect and transfer the complex of data through *Information and Communication Technologies* (Rossi et al. 2024a).

Application

Students were introduced to cubical perspective through a double path (Rossi et al. 2021): first, learning its theory and drawing an imaginary space through exercises of increasing complexity, and second (focused here), linking this knowledge to reality by analyzing the Solimene factory.

We acquired a photographic panorama, then converted it into cubical format (Fig. 10), and synthesized the the building's main components tracing the picture with vector-based tools (Fig. 11, black layer). Next, ideal geodesics were built applying the shortcut and using key points from the ramp and the tree-like columns (Figs. 11, 12). These geodesics were compared with the first drawing (Fig. 11), with the panorama (Fig. 12), and with previous surveys (Fig. 13). We analysed,

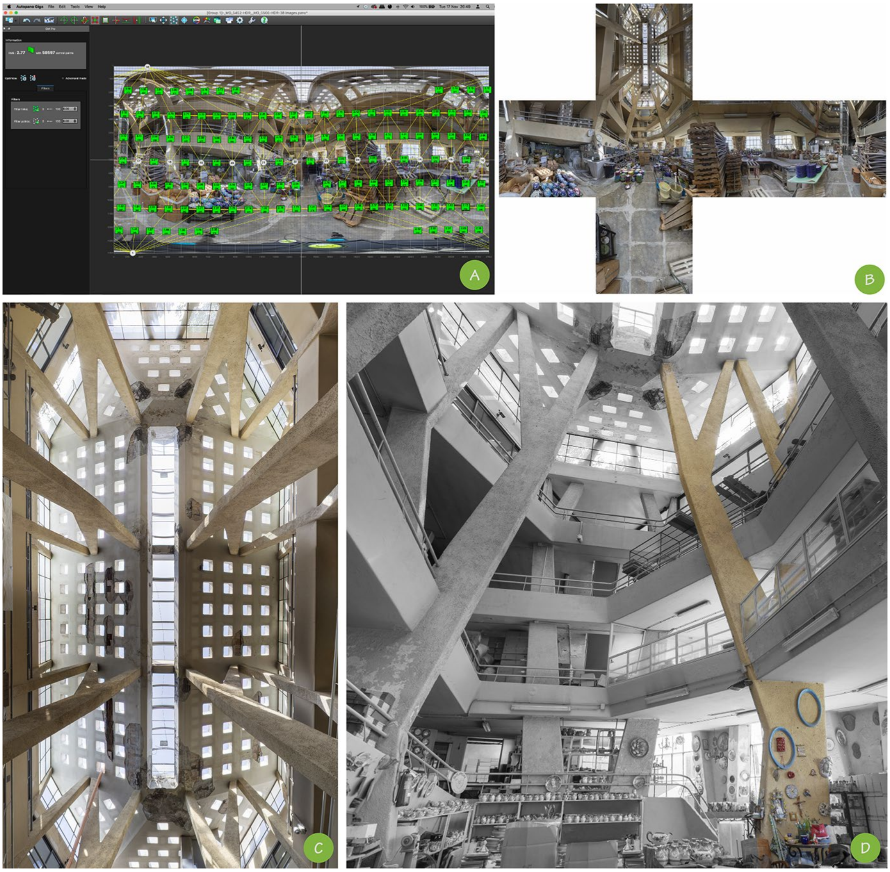

Fig. 10 Panorama's stitching (**A**). Cubical panorama (**B**). Interior details (**C**, **D**)

observed and estimated possible constructive deviations from the column's ideal position (determined by documentation and ideal geodesics) to their most likely position (given by the picture and traced elements) (Figs. 11, 12, 14). Also, occluded columns were reconstructed by interpreting documents, drawing reference blocks, and translating them using geodesics (Figs. 11, 13).

The variety of architectural components provided a cubical composition useful for practicing and testing the shortcut with a full casuistic of geodesics (4-side, 6-side, with points in the same and in different faces, parallel and non-parallel to the cube's faces) using combinations of the first path's techniques and classical perspective operations (repetition/multiplication/subdivision). Finally, students presented interactive virtual tours showing line drawings, coloured scenes, photography, and the critical analysis performed in cubical perspective (Figs. 13, 15).

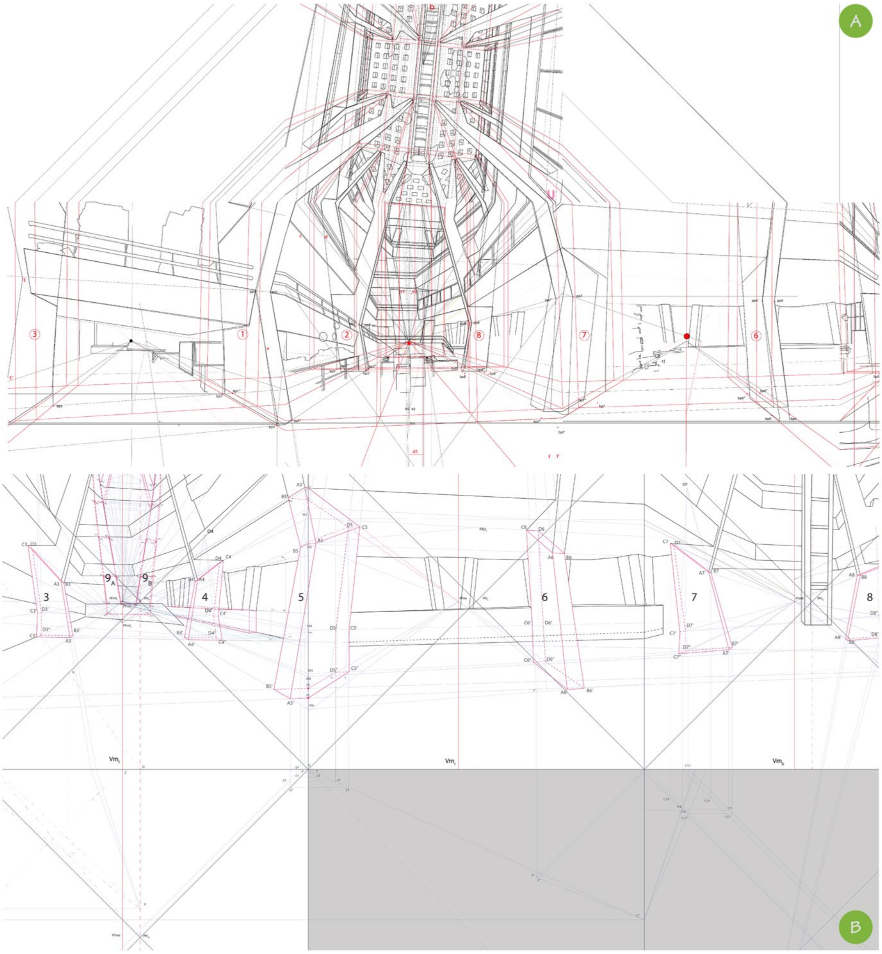

Fig. 11 Comparison showing the first tracing (black) and ideal geodesics built using the shortcut (red). Authors: Groups 1 (**A**), 2 (**B**) (See footnote 3)

Comparison of Methods

The Old Construction builds C' externally, outside the faces of A' and B', requiring descriptive geometry operations onto an external auxiliary plane $\boldsymbol{\delta_e}$. This complicates and slows down the drawing process especially when outside of a studio environment. Instead, the new A-Construction builds C' internally, without further operations on an external plane. This is important because handmade drawing errors propagate with the number of operations and with

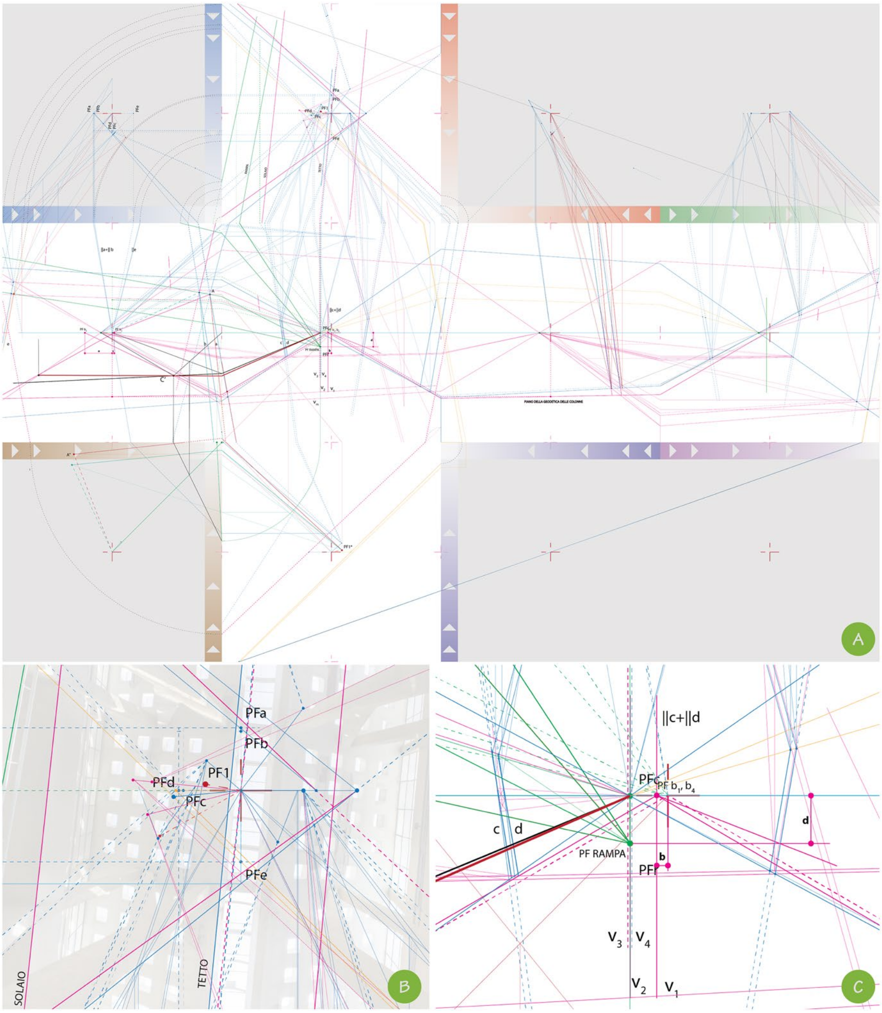

Fig. 12 Ideal geodesics built using the shortcut (**A**). Comparison, ideal geodesics and panoramic picture (**B**). Possible constructive misalignments (**C**)

the crowding of construction lines especially outside of studio conditions. This new solution compresses the scheme to fewer lines condensed in a self-contained outline, reducing the number of steps and improving the user's ability to use cubical perspective for sketching and simplifying/accelerating the studio process. Notice that both constructions can still coexist. When points A' and B' have similar heights, the lines obtained by the Old Construction may touch

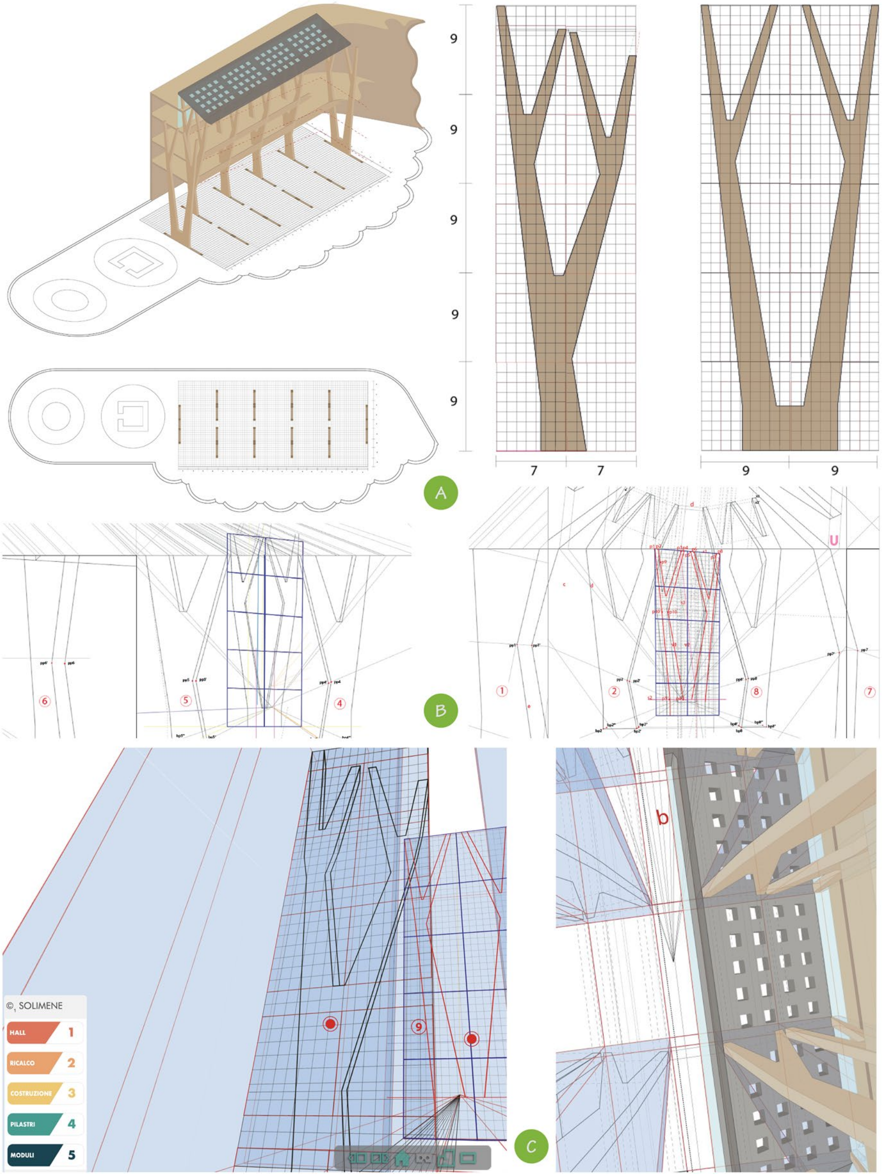

Fig. 13 Analysis of previous surveys' drawings (**A**). Correspondences and reconstructions in cubical format (**B**). Interactive virtual tour (**C**). Authors: Group 1

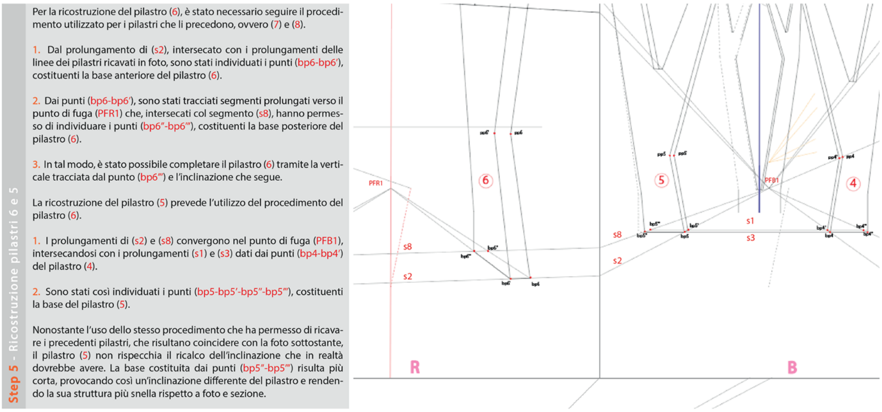

Fig. 14 Report on possible constructive misalignments. Authors: Group 1

at a larger angle than those of the A-Construction, making the plotting of the intersection more precise. Another option is to take antipodes, as described in the discussion of special cases of the A-construction. The more convenient choice may in these cases be left to the illustrator's discretion.

Conclusion

The paper introduced, proved and tested the A-Construction as a new method for rendering the hardest cases of line projections in cubical perspective. The method renders the image of geodesics from two given points and simplifies the method of Araújo et al. (2020) by eliminating the external auxiliary plane. The method assumes point projections with different heights; otherwise, taking antipodes makes it applicable. By applying the new method to the Solimene factory, the case study tested and verified its usefulness and practicality. The A-Construction simplifies cubical perspective drawing, making the practice lighter and more stimulating for both advanced users and neophytes within architectural, design, engineering, and artistic applications.

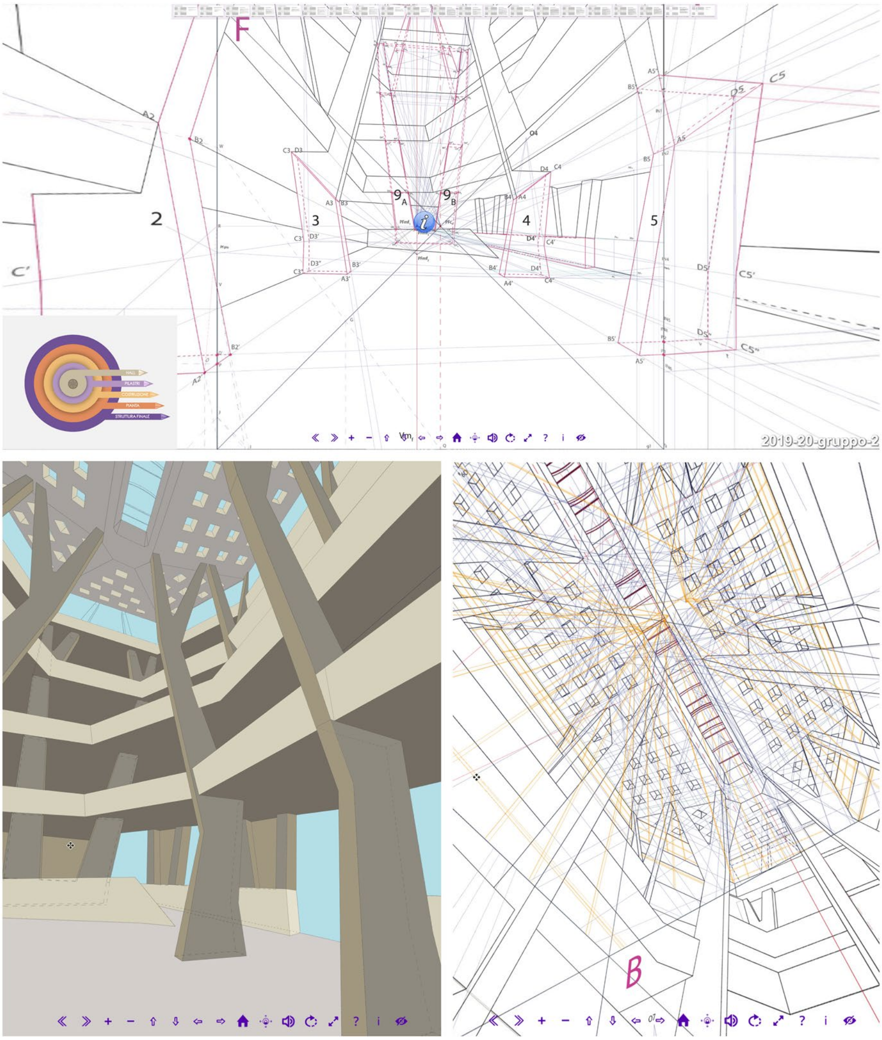

Fig. 15 Virtual tour showing different drawings and analysis. Authors: Group 2

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Declarations

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