

**OPTIMIZED ORBIT TRANSFER OF A GRAVIDYNE WITH LOW-THRUST PROPULSION**

**David S. Gil<sup>ad\*</sup>, Clovis de Matos<sup>b</sup>, Nuno Marques de Sousa<sup>a</sup>, Massimiliano Vasile<sup>c</sup>**

<sup>a</sup> *Universidade Aberta, Portugal*

<sup>b</sup> *European Space Agency, Paris, [Clovis.De.Matos@esa.int](mailto:Clovis.De.Matos@esa.int)*

<sup>c</sup> *University of Strathclyde, Glasgow, United Kingdom*

<sup>d</sup> *National Science and Space Technology Center, United Arab Emirates University.*

\* Corresponding Author [davidgil@uaeu.ac.ae](mailto:davidgil@uaeu.ac.ae)

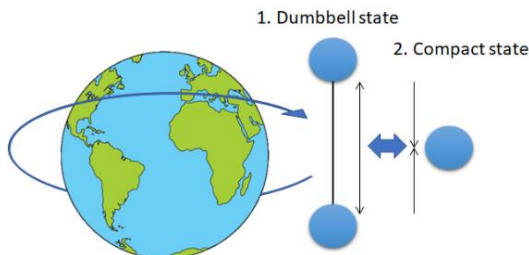
**Abstract**

The Gravidyne devised by Vladimir Beletski is a tethered spacecraft formed by 2 masses orbiting in parallel planes to the spacecraft center of mass. By controlling the distance between the masses, it is possible to increase or decrease the energy of the orbit without the need to spend propellant. However, the Gravidyne needs to be of large dimensions to provide any substantial benefit, which makes it impractical to deploy, hazardous to other spacecraft and in any case, this device alone does not allow to circularize orbit nor change the orbital plane. It is clear that to be practical, the Gravidyne spacecraft needs to use some other type of propulsion. This raises the question of when and where it is more beneficial to use each type of propulsion. This paper presents the results of the optimization of orbital raising and circularizing of a hybrid Gravidyne/low-thrust spacecraft along with conclusions about the benefits of each type of propulsion in each part of the maneuver. A novel secular retardation of the periapsis of the Gravidynes is also presented.

**Keywords:** Gravidyne, Propulsion, Gravity

**1. Introduction**

The Gravidyne is a type of spacecraft devised by Vladimir Beletski [2], it is a tethered spacecraft formed by two masses orbiting in parallel planes, connected by a rigid element of neglectable mass. By controlling the distance between the masses, it is possible to increase or decrease the energy of the orbit of the centre of mass of the Gravidyne without the need to spend propellant.



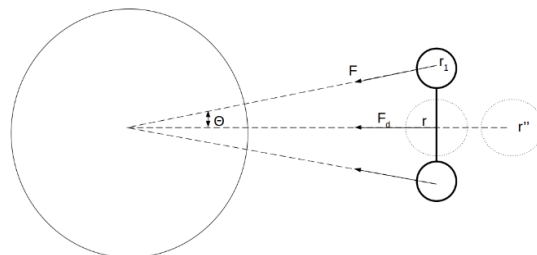
**Figure 1. Gravidyne Dumbbell and Compact States**

*1.1 Compact State*

In the Compact state the Gravidyne is considered a punctual body of mass 2m where the classical orbital description can be applied.

*1.2 Dumbbell State*

In the Dumbbell state it consists of two bodies of mass m united by a rigid cable of length 2l and neglectable mass. The direction of the dispersion of the two masses, e.g., the direction of the rigid cable that unites the masses is perpendicular to the orbital plane. In this configuration, it can be considered that the Gravidyne behaves like if the force of gravity is attenuated.



**Figure 2. Geometry and Notation of the Gravidyne**

As shown in Equation 1 this reduction is greater with the distance between the masses. In the limit of distance l between the two masses tending to infinity, the gravity force applied in the Gravidyne is asymptotically zero.

$$\ddot{\vec{r}} = -\frac{\mu}{r^2} \left( \frac{1}{\sqrt{1+(\frac{l}{r})^2}} \right) \hat{r} \quad (1)$$

Where:  $r$  is the distance to the central body,  $\hat{r}$  the corresponding unitary vector,  $l$  is half of the distance between the masses and  $\mu$  the standard gravitational parameter.

### 1.3 Transition between states

The transition between the two states is considered instantaneous.

## 2. Material and methods

The method used to solve the movement of the Dumbbell (Equation 1) and implement simulations was the Runge-Kutta method of 4<sup>th</sup> order. For the Compact state, an analytical model based in equations derived from Keplerian orbit formulation found in [9,13] were used.

## 3. Theory and calculation

### 3.1 Gaining orbital Energy without reactive propulsion

$$\xi_d = \frac{v^2}{2} - \frac{\mu}{\sqrt{r^2+l^2}} \quad (2)$$

Where:  $\xi_d$  is the specific total energy of the Dumbbell and  $v$  the relative velocity of the Dumbbell to the central body.

The vis-viva equation for the Dumbbell (Equation 2) shows that starting with a circular orbit in the Compact state, the energy increases in case of sudden conversion between Compact to Dumbbell state although the velocity remains the same. Consequently, the equilibrium of forces in the radial axis is disrupted in favour of an increase in radius until the object reaches the apoapsis, where a conversion to Compact state happens and a Gravidyne propulsion cycle is completed.

$$\frac{d\xi_d}{dl} = \frac{l\mu}{(r^2+l^2)^{\frac{3}{2}}} \quad (3)$$

From the derivative of the vis-viva equation with respect to the distance  $l$  between the masses - Equation 3 – we can deduce the following dynamic effects:

- At Periapsis, a conversion from Compact to Dumbbell causes an increase in orbital energy
- At Apoapsis, a conversion from Dumbbell to Compact causes a decrease in orbital energy (the variation of  $l$  is negative)

- However, because, by definition, the radius at Apoapsis is larger than the radius at Periapsis but the distance  $l$  is the same (in absolute value): This means that the loss of energy at periapsis is smaller than the gain.
- Hence after each cycle, the overall orbital energy increases cycle after cycle.

Figure 4 confirms this gain in energy and also a secular drift of the perigee which is also observable in Figure 3.

[1] provides a detailed study of the increase of eccentricity during this pulsating movement.

### 3.2 Secular Drift of the Periapsis

This section provides an analytical modelling of the periapsis secular drift observed during simulations (Figures 3 and 4).

The Lagrangian of the Gravidyne in polar coordinates is given by Equation 4.

$$L = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\alpha}^2) + \frac{\mu m}{\sqrt{r^2+l^2}} \quad (4)$$

Calculating the Lagrange equations, we conclude that the angular momentum is constant:

$$h = mr^2 \dot{\alpha} \quad (5)$$

This is expected in a system where the only applied force is a central force. After simplification for small eccentricity and  $l \ll r$ , we obtain the following equation for the orbit:

$$\frac{1}{r} = \frac{1+e \cos(\chi(\alpha-\alpha_0))}{p} \quad (6)$$

where  $\chi = \left(1 + \frac{3l^2}{p^2}\right)^{-1/2}$ . Equation 6 corresponds to an ellipse whose major axis rotates, at each revolution by an angle (in radians):

$$\delta = -\frac{3\pi l^2}{p^2} \quad (7)$$

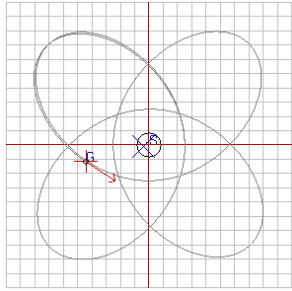
where  $p = a(1 - e^2)$  is the orbital semi-parameter.

## 4. Results

### 4.1 Movement of the Gravidyne, permanently in the Dumbbell State

Figure 3 shows the path of the Gravidyne in Dumbbell state during a number of orbits. In this

simulation the distance between masses used was  $\frac{l}{r} = 0.15$ , which is very large.

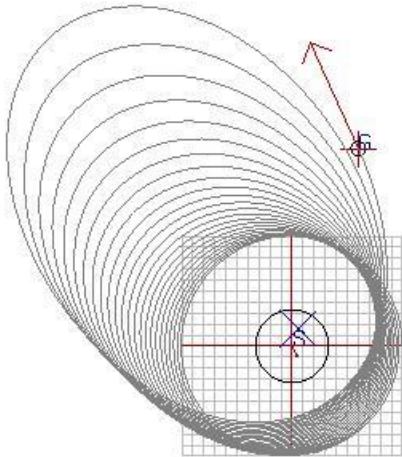


**Figure 3. Orbital path of an extreme Dumbbell**

Unexpectedly, a pronounced drift of the periapsis was observed in this simulation.

**4.2 Movement of the Gravidyne enduring propulsion cycles**

Figure 4 shows the path of the Gravidyne pulsating between Compact and Dumbbell states at periapsis and apoapsis. The used  $l/r$  was 0.05 which is still large for a central body of a planet like Earth and Low Earth Orbits.



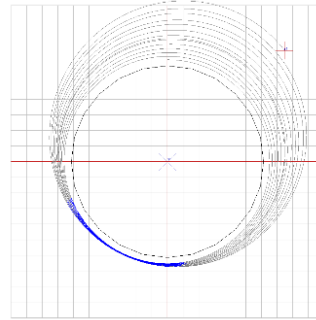
**Figure 4. Orbital path of the Gravidyne enduring propulsion cycles**

It is observed that: (1) even though the periapsis decreases, the semi-major axis increases, hence orbital energy increases (2) eccentricity increase and (3) the periapsis is subject of a secular drift.

**4.3 Movement of the Gravidyne in Compact state with low-thrust propulsion**

Figure 5 shows the path of a punctual mass that features low thrust propulsion. An optimization algorithm was used to decide when to use the propulsion,

targeting a given orbital altitude while trying to minimize propellant usage.

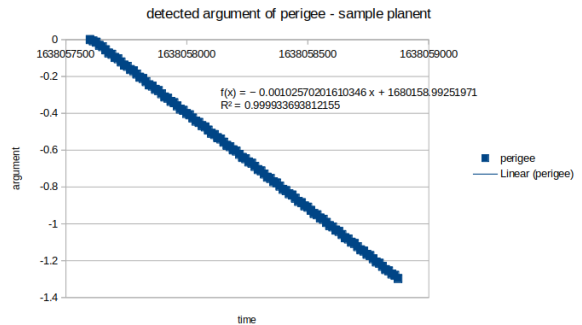


**Figure 5. Orbital Path of a Compact Gravidyne with low thrust propulsion**

Although rudimentary optimization algorithms were used, the result is in line with previous literature [12,14]

**4.3 Characterization of the secular drift of the periapsis**

To verify Equation 9, the first approach was to detect the periapsis orbit after orbit. The results are found in Figure 6.

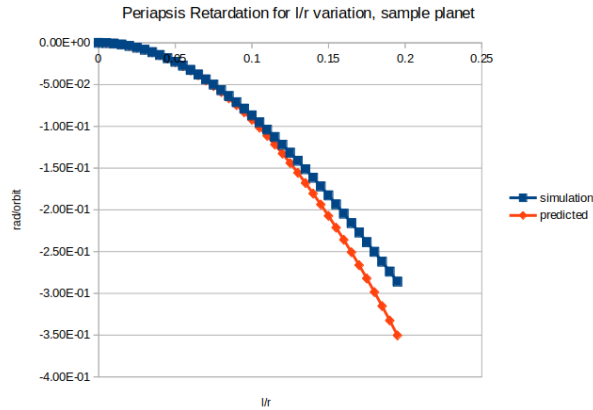


**Figure 6. Drift of detected periapsis with time**

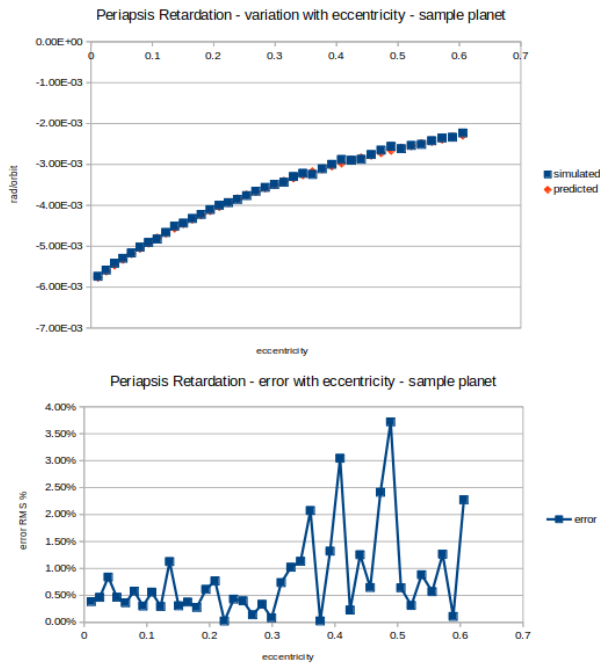
As it can be seen, the drift of the periapsis is linear orbit after orbit with a strong correlation factor. The slope of this drift, obtained by linear regression can be compared and used to validate the method in section 3.2 and Equation 7.

The performance of the model was measured against the two approximations that were used: small eccentricity and  $l \ll p^{3/2}$ .

As it can be seen in Figure 8, that the error for the predicted Periapsis drift was always below 3.5% for even high values of Eccentricity but in Figure 7 we see that the periapsis drift with respect to  $l/r$  only show low errors for  $l \ll p^{3/2}$  (as expected).



**Figure 7. Periapsis drift vs.  $l/r$**



**Figure 8. Predicted vs. detected periapsis for different eccentricities.**

## 6. Conclusions

It was verified by simulation that the Gravidyne gains orbital energy when pulsating between Compact and Dumbbell state as well as it increase in eccentricity. Although a large distance between the masses is needed, the Gravidyne is one of the few types of propulsion that delivers an increasing orbital energy without using propellants.

A novel artifact was discovered: a secular drift in the periapsis. A mathematical model of this drift was proposed and its accuracy was verified and found accurate within the considered constrains: small eccentricity and small distance between masses when compared with the orbital semi-parameter.

While even for large values of eccentricities, the model displays an error never above 3% - which is still low for applications that do not require high accuracy. The model is more sensitive to the constrain of small distance between masses.

## Acknowledgements

Acknowledgement to Mr. Mukesh Jha for helping with general review.

## References

- [1] T. Alhalel, "Un moyen de propulsion surprenant: le gravidyne" Bulletin de l'Union des Physiciens, Vol. 96, (2002).
- [2] V.V. Beletski, "Motion of an artificial satellite about its centre of mass", Science, Moscow, 1965, translated by NASA TT F-429, 1966.
- [3] V.V. Beletski, "Essais sur le mouvement des corps cosmiques", Éditions Mir, 1986.
- [4] R. Lehoucq, "Le gravidyne", Pour la Science, juin 1999.
- [5] M. Martinez-Sanchez and S.A. Gavit, "Orbital modications using forced tether length variations", Journal of Guidance, Control, and Dynamics, Vol. 10, No. 3 (1987), p. 233-241.
- [6] G.A. Landis and F.J. Hrach, "Satellite relocation by tether deployment", Journal of Guidance, Control, and Dynamics, Vol. 14, No. 1 (1991), p. 214-216.
- [7] G.A. Landis, "Reactionless orbital propulsion using tether deployment", Acta Astronautica, 1992, Vol. 26, 5, p. 307-312.
- [8] R. Hoyt, J. Slostad, I. Barnes, N. Voronka and M. Lewis, "Cost-Effective End-of-Mission Disposal of LEO Microsatellites: The Terminator Tape", 24th Annual AIAA/USU Conference of Small Satellites, 2010
- [9] D.A. Vallado, "Fundamentals of Astrodynamics and Applications", Microcosm Press, 2013, ISBN 978-1881883203.
- [10] E. Manor-Chapman, "Cassini Solstice Mission Overview and Science Results", E2013 IEEE Aerospace Conference, 2013.
- [11] S. Kemble, "Interplanetary Mission Analysis and Design", Springer, 2006. ISBN 10 3-540-29913-0.
- [12] M. di Carlo, M. Vasile, S. Kemble, "Optimised GTO-GEO Transfer Using Low-Thrust Propulsion", Springer, 2017.
- [13] T. Kibble, "Classical Mechanics. European Physics Series" (2nd ed.), Mc-Graw Hill, 1973, ISBN 978-0-07-084018-8.11
- [14] F. Zuiani, M. Vasile, "Extended analytical formulas for the perturbed Keplerian motion under a constant control acceleration", Celestial Mechanics and Dynamical Astronomy, 2015, Vol. 121, No. 3 pp. 275-300.