José Sebastião e Silva (1914–1972)

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Introduction

José Sebastião e Silva was the foremost Portuguese mathematician of the 20th century. He was a remarkable man and scientist, with multifarious interests, whose influence in Portuguese society was tremendous and is still felt nowadays.

Born on 12 December 1914 in Mértola (a small town in Portugal’s deep South), he graduated in mathematics in 1937 from the University of Lisbon. After a period of five years when he was essentially unemployed, barely subsisting with private lessons and occasional teaching duties in private schools, he was hired as a teaching assistant at the Faculty of Sciences of the University of Lisbon in 1942 and, in the following year, he went to Rome with a scholarship from the Instituto para a Alta Cultura. He stayed at the Istituto di Alta Matematica until December 1946.

Before leaving for Rome, he was an active member of the Movimento Matemático (Mathematical Movement), an informal gathering of young Portuguese mathematicians that was very important in the promotion of a deep renewal in mathematical teaching and research in Portugal [2]. The group was active from about 1937 until the late 1940s and, during this time, made an important public intervention. At the end of the 1940s, the repressive backlash by the right-wing, dictatorial regime, in the wake of the rigged 1945 general elections, led to the firing from university teaching positions, the imprisonment and the exile of a large number of its members.

In about one decade, the Movimento accomplished several milestones in Portugal’s scientific landscape, such as the creation in 1937 of the scientific journal Portugaliae Mathematica (nowadays published by the EMS Publish-
Sebastião e Silva’s research

Sebastião e Silva’s main research area was functional analysis but his first papers, in 1940, 1941 and 1946, involved a numerical method for the zeros of algebraic equations. Although these were the first research efforts of a young mathematician, their quality and importance is attested by the fact that they were still central in the area of numerical factorisation of polynomials 30 years later [3].

In these early years, Sebastião e Silva also devoted attention to point set topology, then a very active field for the members of the Movimento Matemático.

After moving to Italy, Sebastião e Silva started two lines of work: one in mathematical logic and another in functional analysis.

In functional analysis, Sebastião e Silva worked with the Italian mathematician Luigi Fantappié in the theory of analytic functionals. Sebastião e Silva improved Fantappié’s definition of an analytic functional and, with his new definition, was able to introduce a linear structure on the set of analytic functionals. Next, he introduced the notion of a convergent sequence of analytic functionals. In order to define a topology in the linear space of analytical functionals, he was led to the study of topological vector spaces and, in particular, locally convex spaces. At the same time, he was interested in Laurent Schwartz’s theory of distributions.

In the theory of locally convex spaces, Sebastião e Silva studied special cases of inductive and projective limits, defining the spaces LN* and M*, which are known today as Silva spaces.

In the theory of distributions, Sebastião e Silva introduced an axiomatic construction of finite order distributions. This construction was suggested by a previous model of finite order distributions: they are entities of the form $D^n F$, where $F$ is a continuous function and $n$ is a multi-index. The finite order space of distributions is the quotient space of the Cartesian product of the sets of continuous functions and of non-negative integers by a suitable equivalence relation.

Sebastião e Silva was very proud of his axiomatic construction of distributions. We would like to quote one of his remarks:

“Dans le cas de la théorie des distributions, comme dans d’autres cas, les modèles se sont présentés avant l’axiomatisation. Et c’est justement la pluralité de concepts concrets, ontologiques, de distribution (comme fonctionnelles, comme séries formelles, comme classes de suites de fonctions, comme couples de fonctions analytiques, etc.), qui suggère d’en extraire la forme abstraite, par axiomatisation. Une définition en plus? Oui et non: il s’agit alors de faire une synthèse des définitions “concrètes”; ce qui en résulte sera plutôt la
Still in the theory of distributions, Sebastião e Silva introduced the concepts of limit of a distribution at a point, of order of growth of a distribution and of integral of a distribution. The notion of integral allowed him to write the convolution of distributions by the usual formula for functions; a similar situation occurs with the Fourier transform of distributions.

In some problems of differential equations, there is the need for complex translations; they do not exist in the theory of distributions. Also in quantum mechanics there is the need for multipoles series; these series are not convergent (unless they reduce to a finite sum) in the theory of distributions. These problems led Sebastião e Silva to the study of ultradistributions. In a paper of 1958 [5], he introduced, using the Stieljes transform, two new spaces: the space of tempered ultradistributions and the space of exponential growth ultradistributions. The first one contains the space of tempered distributions and the second one the space of distributions of exponential growth. In these spaces, besides the operators of derivation and of product by polynomials, Sebastião e Silva defined the complex translation operator and proved the convergence of some multipoles series.

Sebastião e Silva also introduced the space of ultradistributions of compact support. It is a subspace of the space of tempered ultradistributions. In this new space, he conjectured a necessary and sufficient condition for a multipoles series to be convergent, the proof of which was completed later by one of his pupils Silva Oliveira.

Sebastião e Silva generalised the Fourier transform to the space of ultradistributions of exponential growth. In that space, this new Fourier transform is a linear and topological endomorphism. This is a beautiful generalisation of Schwartz’s result for tempered distributions.

The last research paper written by Sebastião e Silva, when he was hospitalised with a terminal illness, was published posthumously. It deals with an application of tempered distributions with values in a Hilbert space to the Boltzmann equation [6].

The experience of Modern Mathematics, the name by which this project became known, started in the school year 1963/64 in three class groups, taught in three schools (in Lisbon, Porto and Coimbra) by senior teachers who supervised teacher training as part of the commission’s work. In the following years, the project was progressively broadened to other schools; new teachers were trained and texts and support materials were written both for students and for teachers of the experimental classes. These texts, Compêndio de Matemática² and Guia para a Utilização do Compêndio de Matemática³ [7], were written by Sebastião e Silva and are still reference works for the studying and teaching of mathematics at pre-university level.

Sebastião e Silva was the foremost Portuguese mathematician in the 20th century and also, undoubtedly, the most influential one. As a professor he was a strong leader of mathematics reform, having a pivotal role in the modernisation of mathematics teaching at all levels. As a researcher, he was at the forefront of several fields in analysis and had a brilliant international career, in particular as the author of an alternative route to developing the theory of distributions. His standing as a researcher may be assessed from the number of items he authored: 49 items listed in ZentralBlatt and 45 in MathSciNet, plac-
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References


1 Free translation from the Portuguese by JB.
2 Compendium of Mathematics.